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ABSTRACT

Preliminary results are reported for courses in college algebra and trigonometry based on the Postlethwait audio-tutorial minicourse system which were developed at Indiana University-Purdue University at Fort Wayne (IPFW). Two IPFW faculty members, two secondary teachers, and two undergraduate mathematics majors formed a minicourse project team supported by a National Science Foundation Grant with each team member responsible for authoring a number of minicourses, i.e., writing study guides, designing art work in the study guides, developing the tape scripts and preparing the manuscripts for duplication. Although one individual was responsible for a given minicourse, the end product was a team effort. Instruction was designed to involve students actively in three different types of study sessions--independent study, weekly general assembly sessions, and integrated quiz sessions for small groups. The study guide for the minicourse on exponential functions is included. (JEG)

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# AUDIO-TUTORIAL MINICOURSES FOR COLLEGE ALGEBRA AND TRIGONOMETRY: A PROGRESS REPORT

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ABSTRACT

AUDIO-TUTORIAL MINICOURSES FOR COLLEGE ALGEBRA AND TRIGONOMETRY:
A PROGRESS REPORT

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This paper is a report of the author's work in adapting the Postlethwait audio-tutorial minicourse system of teaching to the college algebra and trigonometry courses offered at Indiana University-Purdue University at Fort Wayne (IPFW). Materials sufficient to teach one-fourth of the course were developed during the summer of 1976 and were used during the spring semester of 1977. A report of these preliminary results will be presented.

The development of the remaining materials has been accomplished in a unique way. During the summer of 1977, two IPFW faculty members, two secondary teachers, and two undergraduate mathematics majors formed a Minicourse Project Team directed by the author. The Minicourse Project Team was supported by a National Science Foundation Grant. Each team member was responsible for authoring a number of minicourses. This included writing the study guides, designing the art work in the study guides, developing the tape scripts, and preparing the manuscripts for duplication. Although one individual was responsible for a given minicourse, the end product was truly a team effort. The interworking of the team members and the unique mix of backgrounds is what truly made the project unique.

The complete series of minicourses will be used by the author during the coming spring semester. The minicourses will be evaluated and revised. Also during the summer of 1978, minicourses for the second course of the algebra-trigonometry sequence will be developed under the NSF grant.

*These activities were supported by a grant from the IPFW Research and Instructional Development Program, a grant from the President's Faculty Awards Program of Purdue University, and the National Science Foundation Grant No. SER 77-02454.

## Introduction

In 1961, Professor S. N. Postlethwait started experimenting with the Audio-Tutorial approach to teaching in an introductory Botany course. The basic premise of his system of instruction is that learning is something done by an individual and is not something done to an individual. The learning activities are structured so that the student is actively involved. The role of the instructor is to create a situation conducive to learning by providing direction, the necessary facilities, and motivation. The system of instruction is designed so that students have successful experiences at frequent intervals. The frequent successes provide motivation for the stuuent. Also, the system allows for the individual differences of the students. [1]

Let us list some of the aspects of an educational system discussed by Postlethwait, Novak, and Murray in [1, pp. 3-8] and see how they apply in a college algebra course.

1. Repetition. The learning of mathematics often requires certain ideas, if not all, to be repeated until they become natural to the student. But, the amount of repetition should be done according to individual needs. Students who know a topic should not be required to participate in the learning activities of repetition whereas students who have not learned a topic might require a number of repetitions. A good educational system would permit students to adjust the amount of repetition to suit their needs.

2. Concentration. Since learning requires the attention of the learner, an educational system should provide a learning environment that is available when the student is alert and receptive. Also, some students can learn by assimilating a lot of information at a given time, others not as much. A good system should accommodate either.

3. Unit Steps. Learning is facilitated when it is broken down into small unit steps. Also, students cannot proceed with understanding unless they know the reason for doing or understand how to do a given operation or algebraic manipulation. It is not enough that one student in the class answer the question posed by the instructor; all should know the answer before proceeding on. Of course this rarely happens in a lecture class. What happens is that the students who do not understand the reason copy down what is done at the board and hope they will understand it later. By continuing with new material in the discussion these students are exposed to new subject matter while not understanding all of the foundations. This could lead to frustration and anxiety and the attitude, "I never could understand math" and/or "I don't care about math, anyway." The system must be structured so that students see that they can learn mathematics.

4. Learning Activities Appropriate to the Objective. If the objective is to have students to learn how to draw the graph of a function, the learning activity should be the drawing of the graph and not the watching of the instructor draw the graph; after all, the instructor probably knows how to draw the graph. Other topics might be best learned by viewing a film or using an electronic calculator.

5. Integrated Learning Experiences. The variety of learning activities must be integrated into a sequence of activities which complete all of the objectives of the unit. For example the viewing of a film on the drawing of a graph might be followed by the student drawing some similar graphs and all followed by the reading of a section of the textbook.

## The Development of the Minicourses

In the Summer of 1976, the development of audio-tutorial minicourses for MA 153, a college algebra course, was started under a Grant for Instructional Development. The work was continued during the 1976-77 academic year under a President's Faculty Awards Grant. Also during this time a proposal was written and submitted to the National Science Foundation Local Course Improvement for Undergraduate Science Education Program. This grant was received in May, 1977. The grant provides the funds for the development during the Summers of 1977 and 1978 of minicourses for MA 153 and MA 154, two of the six algebra and trigonometry courses offered by the Mathematics Department.

The development of the minicourses for MA 153 was accomplished during the Summer of 1977 by a Minicourse Project Team. This team consisted of two undergraduate mathematics majors, two Fort Wayne area secondary school teachers, and two faculty members: Professor Arthur A. Finco, and the author, who served as the project director. Each team member was responsible for authoring a number of minicourses. This included the writing of study guides, the designing of art work for the study guides, the development of tape scripts, and the preparation of manuscripts for duplication. Although one individual was responsible for each minicourse, the end product was truly a team effort. The interaction of team members and the variety of backgrounds are what made the project unique.

## The Audio-Tutorial Minicourse System in Mathematics

Instead of students being passively involved as in a lecture class, they are actively involved in three different types of study sessions: (1) Independent Study Session, (2) General Assembly Session, and (3) Integrated Quiz Session.

The Independent Study Sessions are conducted in the Independent Study Center. Each student individually studies a unit of material called a mini-

course. The students are presented with a variety of learning experiences and the opportunity to determine the rate and intensity of their study within a given week. Students come to the study center with their study guide and textbook, check out the appropriate audio tape, go to a study carrel equipped with a tape player and headphones, open the study guide, and start the tape. The audio tape is not a taped lecture. Instead the audio tape is used as a programming device to guide the students through the various learning activities. In these sessions the students have the option of working independently or with other students, and can request assistance from the instructor or peer tutor on duty if they experience any difficulties. The fact that the students are able to redo any part of a minicourse or even the whole minicourse enables them to repeat the learning activities until they can master the stated objectives of the minicourse.

The General Assembly Sessions are scheduled weekly. They are used to sum up the week's studies in the Independent Study Sessions, to synchronize students studies, and to cover activities best done in a large group. These activities are the showing of long films, the giving of major examinations or help sessions.

In the Integrated Quiz Sessions small groups of students (8-10) meet with the instructor for oral and written quizzes over the week's work. The listed objectives serve as a basis for the quizzes. A significant feature of the quiz sessions is the feedback to the instructor on the success of the materials. A bonus feature of the oral quizzes is that students must learn to express mathematical thoughts orally.

The General Assembly Sessions and the Integrated Quiz Sessions supply student-teacher contact. Also an additional type of student-teacher contact is inherent in the Independent Study Sessions. Here the students are helped

and advised in a one-on-one situation. In addition, a personal contact is simulated because the personality of the instructor is incorporated in the audio tapes.

Deadlines are built into the system. Students must have completed working with the week's material to be prepared for the integrated Quiz Sessions.

## Using the Minicourses

The minicourses were used for the first time in the Spring 1977 semester. At that time minicourses to teach one-fourth of the course had been developed. A comparison of the performance of students taught using the audio-tutorial system of instruction (experimental group) with students taught using the usual lecture method (control group) was made between two sections of MA 153 taught by the author. The students in the two groups were administered five examinations during the semester. The first three examinations covered three units which were taught using the lecture method in both classes. The fourth examination covered a unit taught by the lecture method in one section and the audio-tutorial method in the other section. The fifth examination was a comprehensive examination covering the first three units only.

Each student's performance was measured by the difference between the fourth unit examination and the average score of his four other examinations. The average performance of each student in the experimental group was 9 points better than that of the control group. The statistical likelihood of such a difference occurring at random is less than one percent. These findings, even though limited, show a promise for the audio-tutorial systems of instruction in mathematics.

In Spring, 1978, the author will conduct a much more extensive comparison of two sections of MA 153. One class will again be taught using the lecture-recitation method and the other will use the audio-tutorial system exclusively.

Based on this experience the materials will be revised during the Summer of 1978. Also during the Summer of 1978, minicourses for the second course, MA 154, will be developed under the NSF grant.

The student reaction to the audio-tutorial system of instruction has been very positive. Fifteen students turned in an evaluation of the course in the Spring, 1977 Semester. Fourteen students responded affirmatively when asked, "Would you like to take another whole course using the audio-tutorial system?" and only one responded "no" because his "class schedule does not leave any time to go in." It should be noted, however, that students were scheduled for the lecture style of teaching and had their work schedule arranged according to it. The audio-tutorial style of teaching does not fit the same pattern of scheduling. A whole semester of the audio-tutorial style of teaching where the students arrange their work schedule to fit the system of teaching should eliminate this problem.

The students who responded positively gave as their reasons the following:

> "The audio-tutorial method is much better. If a distraction or misunderstanding occurs, you can simply rewind the tape to hear what you missed."
>
> "It just seems to be easier for me to go at my own speed. The things that I don't understand or don't feel comfortable with I can repeat as often as I need -- which may be many times."
>
> "Sometimes the answer to the question at the board came from someone in the back who probably should be in calculus, because it was hard for me to write down the problem (in keeping up) let alone following what was happening. The audio-tutorial section definitely cut this down considerably. I felt much more at ease in this learning system and if something went by to [sic] fast I could stop and go back through it as many times as needed, then if I did not get it I could ask someone."

Finally, one student responded when asked for suggestions to improve the course, "MAKE AUDIO-TUTORIAL MANDATORY."

In summary, let me ask this question, "What procedure would you use to teach a friend mathematics?" Surely your answer would not be "three lectures a week" or "three lectures and two recitations a week." What you would probably do is meet with your friend at a convenient time and place and have the materials such as graphs, calculators, tables, etc. at hand to discuss the mathematics. Then you would proceed to tutor your friend through a series of learning activities where there would be a one-to-one relationship and your friend would be involved doing calculations, simplifying expressions, or drawing graphs at all times. The Postlethwait Audio-Tutorial System as I have applied it to the college algebra course is designed to simulate this relationship. I make the audio tape for a given minicourse by having all of the required materials at hand and I talk into the tape recorder as if I were tutoring one individual.

Let me describe how I would tutor you through a part of the minicourse on exponential functions. You have before you the study guide which includes the objectives for the minicourse (see Appendix). You have selected the appropiate audio tape. Turning on the tape recorder; you hear the following:

> (Music) This is the minicourse on Exponential Functions. Whenever you hear some of the music that started this tape, stop the tape recorder and follow the instructions given to you at that time. Of course, when you are done with the exercise, start the tape up again. Try stopping and starting the tape.
>
> (Music) Now turn to the top of page 1. Here we have an organism that we have just caught on the surface of Mars. It's a very fast growing organism, and we start with just a little drop. We're told that the organism is given plenty of food (presumably that came from Mars, also). Also, we're told that the organism grows so fast that the amount doubles every second. The question we want to ask is: How long do you think it would take to obtain a kilogram, that's a thousand grams, of this organism? The kilogram obtained would fit in that beaker pictured in Figure 1 and overflow it. O.K., ask yourself, "How long do you think it would take for the organism to grow from the drop until it overflowed that beaker?" Circle what you would guess to be the correct answer and we'll return to this discussion at the end of the minicourse to actually

find out the correct answer. (Music, a signal to stop the tape recorder and follow the given instructions. When students complete the exercise they start the tape again.)

You have in your study guide the following choices:

a. 3 hours b. 1 hour c. 3 minutes d. 1 minute

e. 30 seconds f. 10 seconds g. 2 seconds

After making a choice, you are instructed to turn to page 2 in the study guide. Page 2 of the study guide is below.

-2-

The situation described on the previous page is an example of exponential growth. We shall consider exponential functions and discover the nature of these functions by studying their graphs.

For any real number b where $b > 0$, we call the function f defined by

$$f(x) = b^x$$

an exponential function with base b.

EXERCISE 1: What is the nature of the graph of $f(x) = b^x$, when $b > 1$?

| x | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y = f(x) = 2^x$ | | | | | | | |

Figure 2

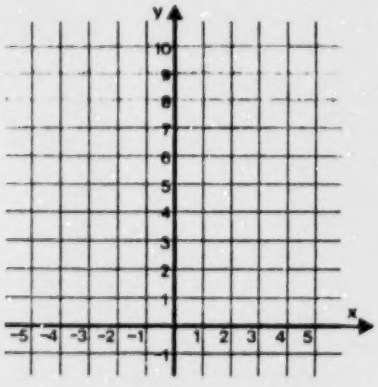


Figure 3

What are some other points on the graph? Is the curve smooth? Let us determine another point on the graph of $f(x) = 2^x$. Let $x = 1.5$. Then $f(1.5) = 2^{1.5}$. Use your calculator to do this calculation. You will need to use the exponentiation key, usually labeled "$y^x$", to find $2^{1.5}$. We have

$2^{1.5} =$ _____.

The definition at the top of page 2 of the study guide is discussed on the tape, and then you are instructed to move to Exercise 1. You hear the following:

> In exercise 1 we are going to investigate the nature of an exponential function f where f(x) equals b to the x and where b is bigger than 1. To do this, we shall first make a table of values for the exponential function f where $f(x) = 2^x$. Pick up a pencil. Now if x equals 3 then 2 raised to the third power is what number? Enter that number in the table in Figure 2. Now suppose we let x equal 0. Then 2 raised to the 0 power is what number? It's one, isn't it? Enter one in the table also. Now suppose x equals minus 3, then 2 raised to the -3 power is what number? Enter that in the table also. Did you enter 1/8? If you did you're on the right track. Now fill out the rest of the table in Figure 2.
> (Music) Now that you have the table filled in, you can check your answers at the end of the minicourse on page 9 listed under item 1. (Music) (The answers to all exercises in the study guide are included at the back of the study guide).

After discussing what some of the other points on the graph would be, you are asked to sketch the curve lying on the points that have been plotted. The nature of the graph is discussed. In other exercises you are instructed how to use the graph of exponential functions to find a one-place decimal approximation for $2^{\sqrt{2}}$, and also how to estimate the solution of equations such as $2^x = 5$. Other graphs are drawn and discussed in the remainder of the minicourse. Always the audio tape is tutoring and guiding you through the exercises and leading you to discover the relevant mathematical principles.

Finally, you return to the discussion of the exponential growth of the organism from Mars that introduced the minicourse. A discussion leads you to discover the exponential function that gives the amount of the organism as a function of time. It is pointed out that the equation needed to answer the question of how long it would take to obtain 1000 kilograms of the organism cannot be solved with the techniques at hand; but the answer is given, and you

are told that you will learn to solve such equations in a later minicourse. Last, you are asked to read some examples from the text and are given an assignment in the text.

## References

1. Postlethwait, S.N., Novak, J. and Murray, Jr., H.T., The Audio-Tutorial Approach to Learning, Third Edition, Burgess Publishing Co., 1972.

APPENDIX

# EXPONENTIAL FUNCTIONS

A minicourse prepared by
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Fort Wayne

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## Objectives for the Minicourse on Exponential Functions

After completing the minicourse you should be able to do the following:

1. Sketch the graph of the function f where $f(x) = b^x$ where b is a rational number greater than 1.
2. Sketch the graph of the function f where $f(x) = b^x$ where b is a positive rational number less than 1.
3. Use the graph of $f(x) = b^x$ to approximate $b^{\sqrt{3}}$, $b^{\pi}$, etc., and to solve an equation such as $b^x = 5$ with an error of at most $\pm .5$.
4. Describe the relationship of the graph of the functions $f(x) = a^x$ and $g(x) = b^x$ where $1 < a < b$.
5. Draw the graph of the function f where $f(x) = e^x$.
6. Find the exponential relationship that describes a growth or decay phenomenon and compute the amount present after some period of time.

Suppose we have a fast growing organism that we have just caught on the surface of Mars and it is able to double its amount every second. If we start with a small drop of the organism, about $\frac{1}{10}$ gram, and have plenty of food for the organism, about how long do you think it would take to obtain a kilogram (1000 grams or a little more than a quart) of the organism? (Circle the answer you guess is closest to the time it would take.)

a. 3 hours

b. 1 hour

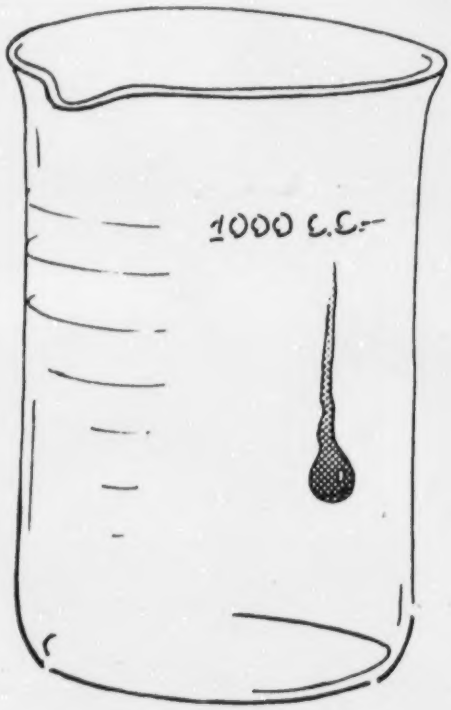
c. 3 minutes

d. 1 minute

e. 30 seconds

f. 10 seconds

g. 2 seconds




From this drop to this amount in how much time?

Figure 1

...IF OUR POPULATION DOUBLES EVERY FOUR YEARS
AND EVERY FAMILY HAS A HORSE AND WAGON...
HOW LONG WILL IT BE BEFORE OUR STREETS ARE FULL?
OF WHAT?

The situation described on the previous page is an example of exponential growth. We shall consider exponential functions and discover the nature of these functions by studying their graphs.

For any real number b where $b > 0$, we call the function f defined by

$$f(x) = b^x$$

an <u>exponential function</u> with base b.

<u>EXERCISE 1</u>: What is the nature of the graph of $f(x) = b^x$, when $b > 1$?

| x | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y = f(x) = 2^x$ | | | | | | | |

Figure 2

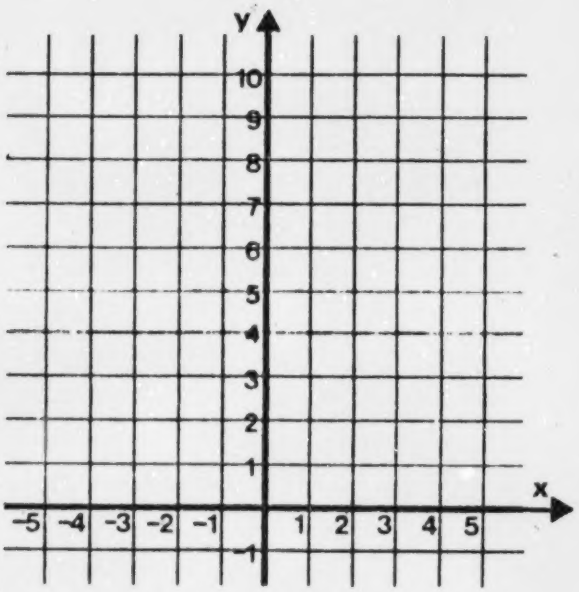



Figure 3

What are some other points on the graph? Is the curve smooth? Let us determine another point on the graph of $f(x) = 2^x$. Let $x = 1.5$. Then $f(1.5) = 2^{1.5}$. Use your calculator to do this calculation. You will need to use the exponentiation key, usually labeled "$y^x$", to find $2^{1.5}$. We have

$$2^{1.5} = \_\_\_\_\_.$$

Plot the point (1.5, 2.8) on the graph in Figure 3 and notice that it just "fills in the hole" of a smooth curve.

This graph is typical of exponential functions with base b where $b > 1$. The graph will always

(1) have y-intercept 1, since $b^0 = 1$,

(2) be increasing,

(3) approach the x-axis asymptotically as $x \to -\infty$, and

(4) rise very rapidly as x increases as $x \longrightarrow +\infty$.

EXERCISE 2: What is the value of $2^{\sqrt{3}}$?

Use the graph of the function $f(x) = 2^x$ in Figure 4 to estimate $2^{\sqrt{3}}$ (recall $\sqrt{3} = 1.73$). We see $2^{\sqrt{3}}$ is approximately 3.3.

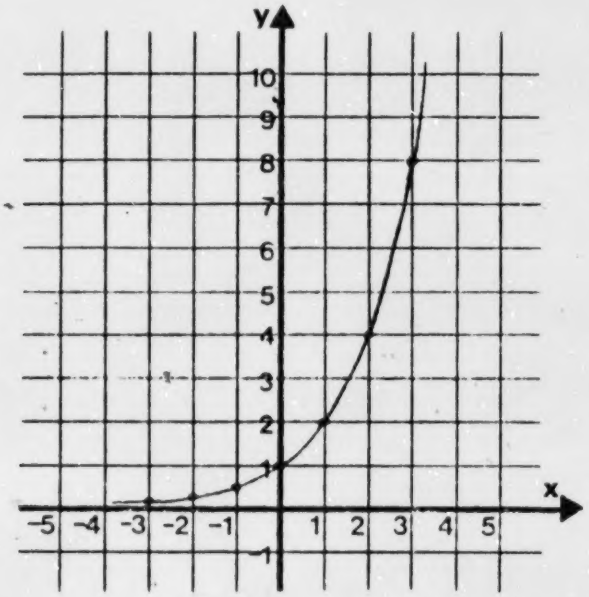



Figure 4

| x | $\sqrt{2} = 1.41$ | $\sqrt{5} = 2.24$ | $\pi = 3.14$ | $-\sqrt{2} = -1.41$ |
|---|---|---|---|---|
| $y = 2^x$ | $2^{\sqrt{2}} =$ | $2^{\sqrt{5}} =$ | $2^{\pi} =$ | $2^{-\sqrt{2}} =$ |

Figure 5

Use your calculator to check the accuracy of your results.

EXERCISE 3: How can we find an approximation for the solution of an exponential equation?

We all know the solution set for the equation

$$2^x = 8$$

is {3} since we know $2^3 = 8$.

Use the graph of the function $f(x) = 2^x$ in Figure 4 to solve the equations in the table in Figure 6.

| equation | solution |
|---|---|
| $2^x = 8$ | x = 3 |
| $2^x = 5$ | x = |
| $2^x = .8$ | x = |
| $2^x = -4$ | x = |

Figure 6

EXERCISE 4: What is the nature of the graph of $f(x) = b^x$, where $0 < b < 1$?

| x | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $f(x) = (\frac{1}{2})^x$ | | | | | | | |

Figure 7

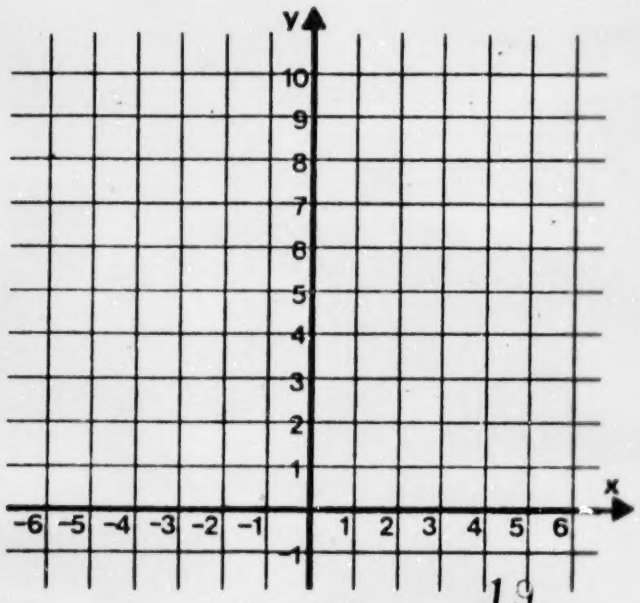



Figure 8

This graph is typical of exponential functions with base b where $0 < b < 1$. The graph will always

(1) have y-intercept _____,

(2) be _____,

(3) approach the x-axis asymptotically as x _____,

(4) rise very rapidly as x _____.

Notice that $(\frac{1}{2})^x = 2^{-x}$ and so the graph is the same as the graph of the equation $y = 2^{-x}$.

EXERCISE 5: How do the graphs of exponential functions with different bases compare with one another?

| x | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y = f(x) = 2^x$ | | | | | | | |

Figure 9

| x | -3 | -2 | -1 | 0 | 1 | 2 |
|---|---|---|---|---|---|---|
| $y = g(x) = 3^x$ | | | | | | |

Figure 10

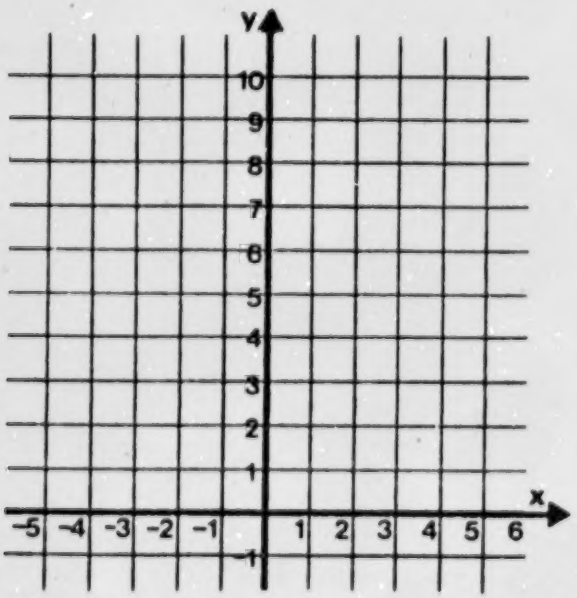



Figure 11

In general if $1 < a < b$, then

(1) the graph of $f(x) = a^x$ lies (below, above) the graph of $g(x) = b^x$ for $x > 0$ since $a^x < b^x$ when $x > 0$, and

(2) the graph of $f(x) = a^x$ lies (below, above) the graph of $g(x) = b^x$ for $x < 0$ since $a^x > b^x$ when $x < 0$.

Now suppose $1 < a < b < c$, and the graphs of $f(x) = a^x$ and $g(x) = c^x$ are as indicated in Figure 12. From the discussion above, where would the graph of $h(x) = b^x$ lie? Sketch the graph of $h(x) = b^x$ in Figure 12.

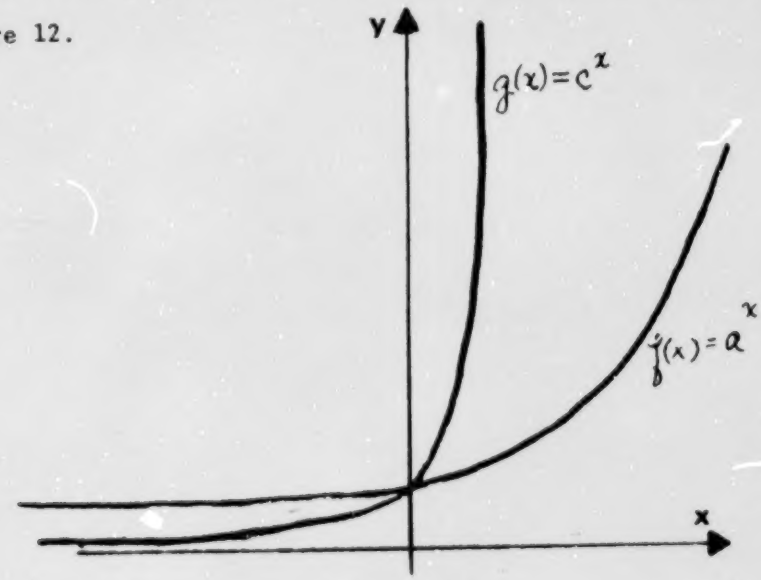



Figure 12

EXERCISE 6: What is the exponential function?

There is a special irrational number called e whose decimal representation to five places is 2.71828. We see that $2 < e < 3$. From these inequalities and our discussion above, we can place the proper inequality, $<$ or $>$, in the table in Figure 13.

| | | | |
|---|---|---|---|
| $e^x$ | $3^x$ | for | $x > 0$ |
| $e^x$ | $3^x$ | for | $x < 0$ |
| $e^x$ | $2^x$ | for | $x > 0$ |
| $e^x$ | $2^x$ | for | $x < 0$ |

Figure 13

Now sketch the graph of the function

$$k(x) = e^x$$

on the graph in Figure 14.

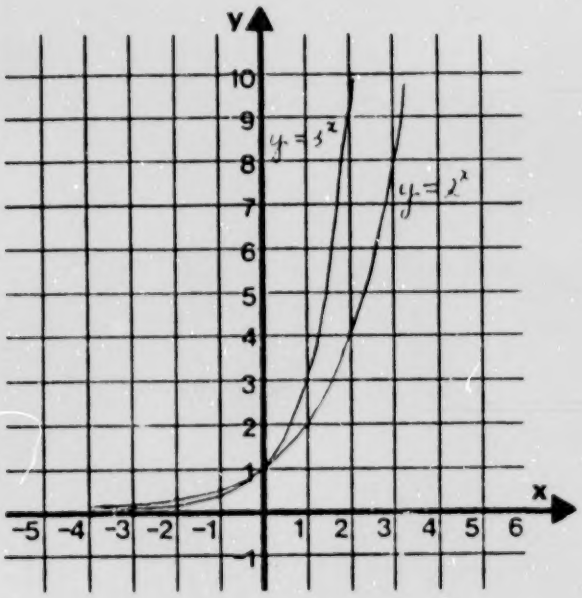



Figure 14

Fill in the table in Figure 15 using the graph in Figure 14.

| x | -3 | -2 | -1 | 0 | 1 | 2 |
|---|---|---|---|---|---|---|
| $e^x$ | | | | | | |

Figure 15

EXERCISE 7: What is the equation that gives the amount A of the organism (discussed at the beginning of the minicourse) as a function of time, t?

$A(0) = \frac{1}{10}$

$A(1) = \frac{1}{10} \cdot 2 =$

$A(2) =$

$A(3) =$

$A(4) =$

Thus

$A(t) =$

To find the time it would take to obtain 1000 grams of the virus we would have $A(t) = 1000$ and we must solve the equation

$$1000 = \frac{1}{10}\, 2^t.$$

In the next minicourse we shall learn the technique to solve this equation and we shall find that

$$t = 13.29.$$

EXERCISE 8: Read, starting at the top of page 161, the examples on pages 161, 162, and 163 of the text.

Assignment: p. 163, #1, 2, 3, 9, 17, 19, 24, and the following problems. Use the graph of $y = 2^x$ in Figure 4 in the minicourse to find approximations for u in the following:

a) Find $u = 2^{\sqrt{3}}$.

b) Find $u = 2^{\sqrt{5}}$.

c) Find $u = \sqrt{2} = 2^{\frac{1}{2}}$.

d) Find $u = \sqrt[3]{2}$.

e) Solve the equation $2^u = 7$.

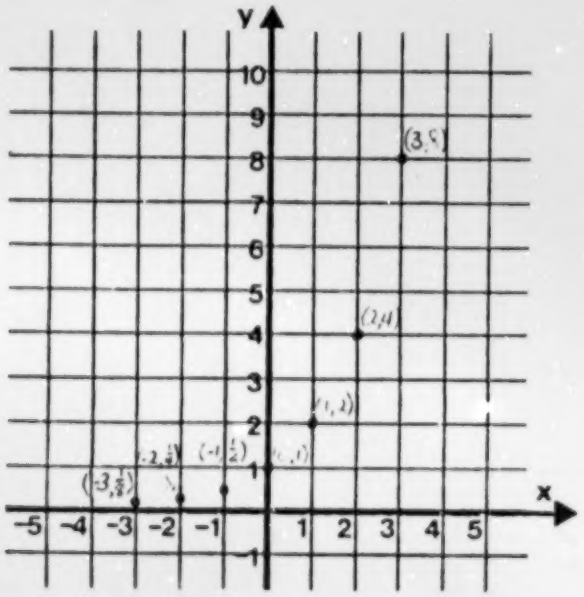
f) Solve the equation $2^u = \frac{1}{5}$.

## Answers for minicourse

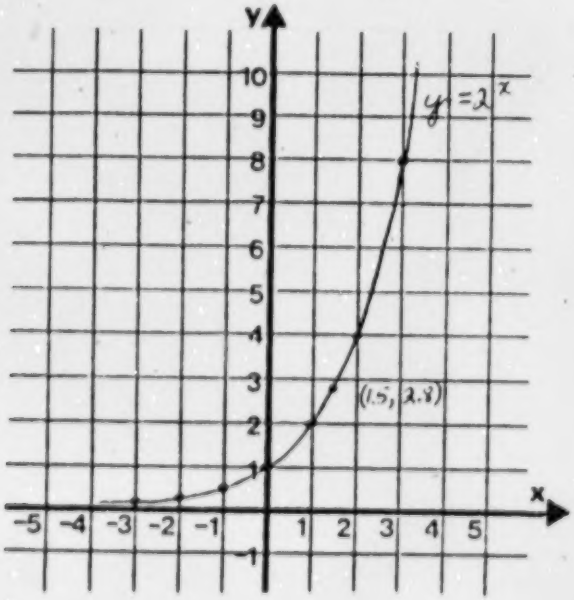
1.

| x | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $f(x) = 2^x$ | $\frac{1}{8}$ | $\frac{1}{4}$ | $\frac{1}{2}$ | 1 | 2 | 4 | 8 |

2.




3.




4.

| x | $\sqrt{2}$ | $\sqrt{5}$ | $\pi$ | $-\sqrt{2}$ |
|---|---|---|---|---|
| $2^x$ | 2.7 | 4.7 | 8.8 | .4 |

5.

| equation | solution |
|---|---|
| $2^x = 8$ | $x = 3$ |
| $2^x = 5$ | $x = 2.3$ |
| $2^x = .8$ | $x = -.3$ |
| $2^x = -4$ | $x =$ (none) |

6.

| x | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $f(x) = (\frac{1}{2})^x$ | 8 | 4 | 2 | 1 | $\frac{1}{2}$ | $\frac{1}{4}$ | $\frac{1}{8}$ |

7.

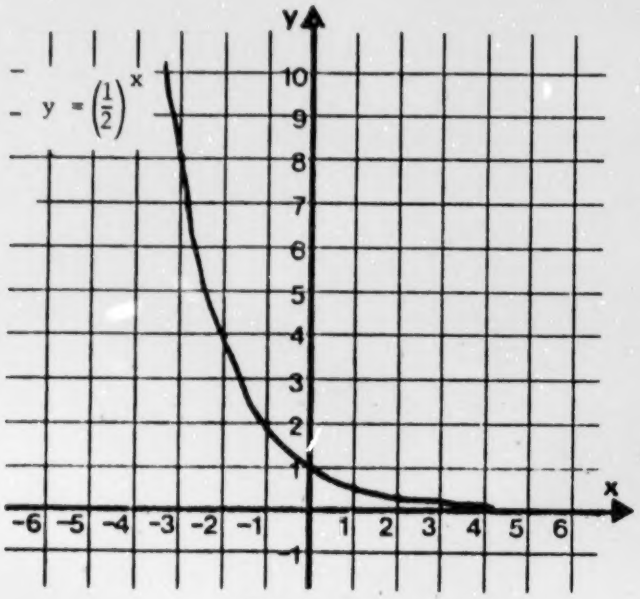



Figure 8

8. (1) one

(2) decreasing

(3) increases as $x \to +\infty$

(4) decreases as $x \to -\infty$

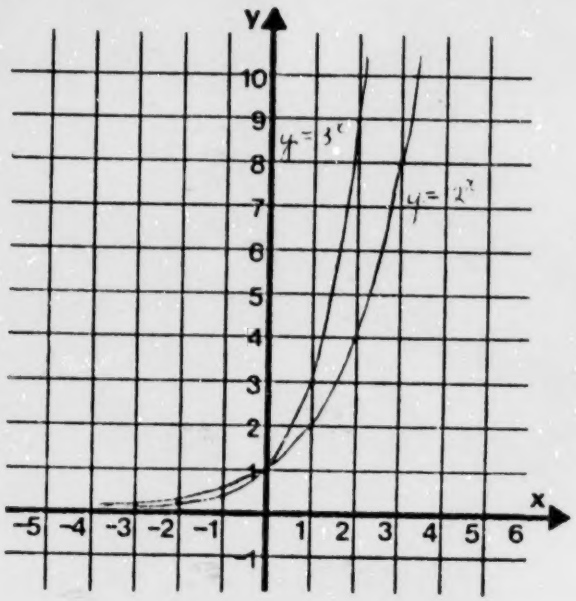
9.

| x | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $f(x) = 2^x$ | $\frac{1}{8}$ | $\frac{1}{4}$ | $\frac{1}{2}$ | 1 | 2 | 4 | 8 |

10.

| x | -3 | -2 | -1 | 0 | 1 | 2 |
|---|---|---|---|---|---|---|
| $g(x) = 3^x$ | $\frac{1}{27}$ | $\frac{1}{9}$ | $\frac{1}{3}$ | 1 | 3 | 9 |

11.




12. (1) below

(2) above

13.

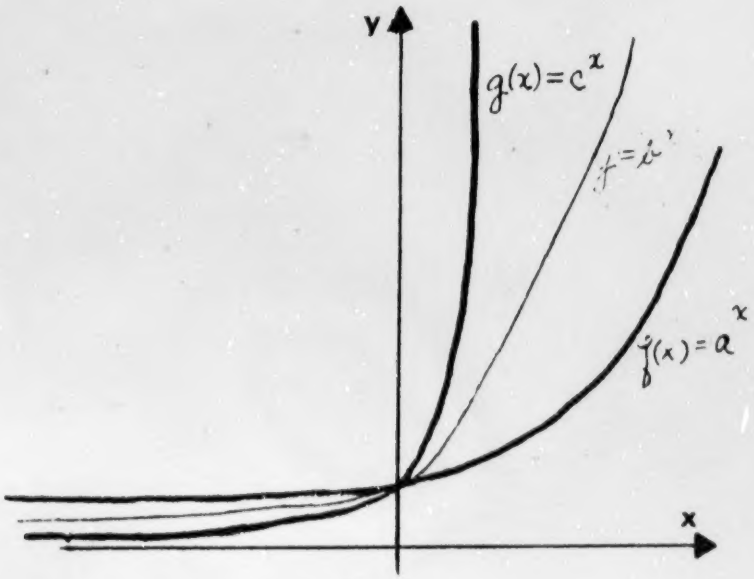



Figure 12

14. $e^x < 3^x$ for $x > 0$

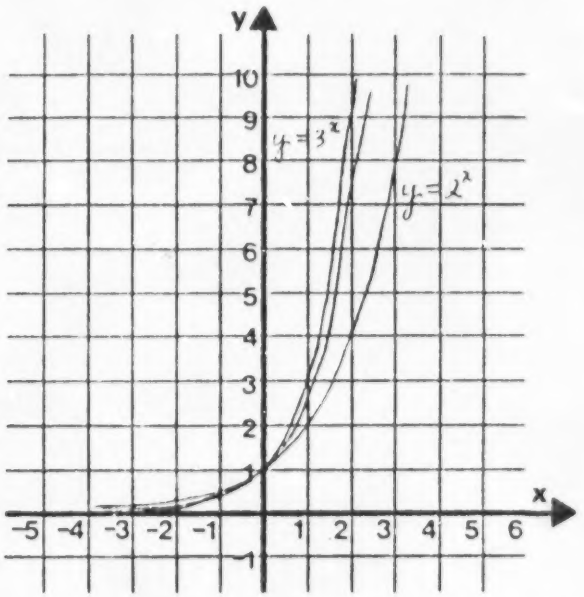
$e^x > 3^x$ for $x < 0$

$e^x > 2^x$ for $x > 0$

$e^x < 2^x$ for $x < 0$

---

15.

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16.

| x | -3 | -2 | -1 | 0 | 1 | 2 |
|---|---|---|---|---|---|---|
| $e^x$ | .05 | .1 | .4 | 1 | 2.7 | 7.4 |

---

17. $A(0) = \frac{1}{10}$

$A(1) = \frac{1}{10} \cdot 2 = \frac{1}{10} 2^1$

$A(2) = (\frac{1}{10} \cdot 2)2 = \frac{1}{10} \cdot 2^2$

$A(3) = (\frac{1}{10} \cdot 2^2)2 = \frac{1}{10} 2^3$

$A(4) = \frac{1}{10} 2^4$